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THE EFFECTS OF PARTICLES ON THE STABILITY OF A TWO-PHASE WAKE FLOW

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Abstract—Analytical and numerical investigations of the effects of particles on the stability of two-phase wake flows are presented. The major simplifying assumptions are that the mean velocity profile of the two-phase flow is identical to that of the single-phase flow and that the fluctuations of the flow field have no effect on the particle initial velocity. The continuous-phase flow is assumed incompressible and inviscid. The resulting modified Rayleigh equation was solved numerically. The results presented are applicable to dilute two-phase flows consisting of solid particles or liquid drops in a gaseous environment. For the two-phase wake flow, it is found that the presence of the particles enhances the stability of the flow. For the absolutely unstable single-phase flow, the particles lower the imaginary part of the branch point and can transfer the flow to the convectively unstable region. For the convectively unstable flow, the particles can lower the most amplified rates in spatial instability.

Key Words: two-phase flow, stability, wake

1. INTRODUCTION

Stability analyses and experiments for single-phase parallel flows, such as mixing layers, jets and wakes, have been under intensive development for the past few decades (Ho & Huerre 1984; Huerre & Monkewitz 1990). Classical hydrodynamic instability theory usually addresses the problem from either the spatial or temporal point of view. In the temporal stability analysis, it is implicitly assumed that the disturbances evolve in time from some initial spatial distribution. The wavenumber is assumed to be real and the complex frequency, which is a function of the wavenumber, is evaluated to determine the stability of the system. The temporal theory works well for the Taylor–Couette and Rayleigh–Benard types of flows. For free-shear layers and boundary layers the instability is usually controlled by periodically forcing the flow at a given frequency. Experimental results seem to correlate closely with the predictions of spatial theory, where the frequency is real and the wavenumber is complex. One important question arises; i.e. how to choose the spatial and temporal theory for a particular flow system. According to Huerre & Monkewitz (1985), a choice can only be made once the absolute or convective nature of the instability has been determined. The concept of absolute vs convective instability was introduced initially by plasma physicists (Briggs 1964; Lifshitz & Pitaevskii 1981; Bers 1975). Recent reviews of the topic for free-shear flows are given in Huerre (1987) and Huerre & Monkewitz (1990). A flow is convectively unstable if its impulse response decays to zero for large times at all points in the flow. In absolutely unstable flows the presence of a transient disturbance at any location leads, in the linear regime, to exponential growth everwhere in the system. In the absolutely unstable case any transients generated by switching on the excitation or any residual background fluctuations will amplify and contaminate the entire flow. Whereas in the convectively unstable case these transients are convected downstream and, thus, spatially growing waves at the excitation frequency can be observed. Hence the spatial theory prediction is only meaningful in convectively unstable flows. Single-phase instability for two-dimensional wake flows has been investigated quite extensively (Mattingly & Criminale 1972; Drazin & Reid 1981). Only recently, Monkewitz (1988) investigated the nature of the instability in two-dimensional wakes at low Reynolds numbers. Monkewitz (1988) found that in the wake both absolute and convective instabilities could exist simultaneously in different regions of the flow, unlike the two-dimensional mixing layer where the flow is only convectively unstable and a spatial stability theory is applicable.

Two-phase (gas–solid particle, gas–droplet and liquid–gas bubble) shear flows can be found in many industrial and energy-related processes (Chigier 1981). The stability of the two-phase flow is important to the design and efficient operation of the entire flow system. The main purpose of the present investigation is to examine the effects of particles on the absolute/convective instability of a two-phase wake flow.

Before addressing the two-phase stability of a wake, one needs to address the interactive dynamics between the particles and the large-scale structures which dominate the wake flows. A physical model based on the mechanics of the large-scale vortices and their interactions with the particles has been developed by Crowe *et al.* (1985) and Chung & Troutt (1988). This model has been verified by numerical simulations (Crowe *et al.* 1985; Chung & Troutt 1988) and experimental results (Kamalu *et al.* 1989; Wen *et al.* 1988). The central idea of this model is that the particle dispersion pattern in flows with large-scale vortex structures is dominated by the ratio of the particle aerodynamic response time to the flow characteristic time associated with the large-scale vortex structures. This time scale ratio is usually referred to as the Stokes number (St). At $\mathrm{St} \ll 1$, solid particles dispersing in gaseous flows will follow closely the flow structure and disperse at the rate of the fluid particles. Particles would be little affected by the flow if the St is large compared to unity. Particles with St of the order of unity, however, might be expected to disperse at rates greater than the fluid particles. This larger dispersion effect is due to the vortical nature of the large-scale structures which create a centrifuging effect to remove the particles from the vortex cores. These recent findings concerning the importance of vortex structure on the partial dispersion process impress the need for multiphase stability analyses, since large vortex structures are probably closely connected to stability type processes.

The stability characteristics of two-phase flows have not received much attention in the literature. Pioneering contributions on two-phase stability have been made by Acrivos & his coworkers (Herbolzheimer 1983; Shaqfeh & Acrivos 1987; Borhan & Acrivos 1988) for sedimentation flows inside an inclined settler. They developed linear stability models for a wide range of system parameters to predict the effects of particles on the stability characteristics during sedimentation. Saffman (1962) discussed the linear stability of a dusty gas. He suggested a possible destabilizing effect due to the decrease in the equivalent kinematic viscosity of the gas and a stabilizing effect due to the friction caused by the particles. Yang *et al.* (1990) investigated the spatial instability of a two-phase mixing layer. In this paper, a simplified mathematical model is introduced as a first step toward understanding the effects of foreign particles on the stability characteristics of the wake flow. The major assumption involved is that the mean velocity profile of the two-phase flow is approximated as that of the particle-free single-phase flow. Laser velocimetry measurements downstream of a splitter plate (Wen *et al.* 1988) showed that the mean velocity profiles at various downstream locations remain similar to those of single-phase mixing layers for particle loading ratios up to 20%. They also found that the mean velocity profile of the particles resembles closely that of the fluid at each downstream measurement location, except at the first few stations in the vicinity of the splitter plate. Therefore, the assumption of dynamic equilibrium between the two phases appears justified.

Section 2 of this paper describes the mathematical model and governing equations. The numerical procedure is briefly outlined in section 3, and the calculated results are discussed in section 4. In section 5, some conclusions from the research are provided.

2. BASIC THEORY AND THE MATHEMATICAL MODEL

General Theory

Before discussing the stability of a two-phase wake flow, one must first review the stability mechanism of particle-free flows.

The classical linear stability theory of parallel shear flows is concerned with the development in space and time of infinitesimal perturbations imposed on a mean flow $U(y)$. The mean flow is a parallel flow in the x-direction.

The linear stability analysis begins be decomposing the velocities and pressure into the mean and transient response components. In the case of two-dimensional flows, the instantaneous velocities and pressure can be written as

$$\tilde{u} = U(y) + u'(x, y, t), \qquad [1a]$$

$$\tilde{v} = v'(x, y, t) \qquad [1b]$$

and

$$\tilde{p} = P_0 + p'(x, y, t). \qquad [1c]$$

Substituting the above into the Navier–Stokes equations and keeping only the linear terms, one obtains a set of linearized perturbed Navier–Stokes equations. The set is solved for the perturbed velocities, which are represented by the two-dimensional stream function $\psi(x, y, t)$. It is assumed that the stream function has the following wave-like form:

$$\psi(x, y, t) = \phi(y)\exp[i(\alpha x - \omega t)], \qquad [2]$$

where α is the wavenumber and ω is the angular frequency. Substituting the stream function into the perturbed Navier–Stokes equation (Drazin & Reid 1981) yields, in most cases, an ordinary differential equation. Enforcement of the appropriate boundary conditions then leads to an eigenvalue problem, whereby the eigenfunction $\phi(y)$ exists only if α and ω satisfy a dispersion equation of the form

$$D[\alpha, \omega, R] = 0, \qquad [3]$$

where R is a system-related parameter.

The solution of this eigenvalue problem, for a certain type of mean velocity $U(y)$, may be viewed as providing a relation $\omega(\alpha)$ between the complex wavenumber α and the complex angular frequency ω.

One can associate a differential or integro-differential operator $D[-i(\partial/\partial x), i(\partial/\partial x), R]$ in the physical space (x, t) with the dispersion [3] in the spectral space (α, ω), such that the stream function $\phi(x, t)$ satisfies the following equation:

$$D\left[-i\frac{\partial}{\partial x}, i\frac{\partial}{\partial t}; R\right]\psi(x, t) = 0. \qquad [4]$$

For the mathematical framework to be outlined next, the readers are referred to Huerre (1987) for more details.

In order to study the absolute/convective instability, one defines the Green's function $G(x, t)$, i.e. the impulse response of the flow, as the following:

$$D\left[-i\frac{\partial}{\partial x}, i\frac{\partial}{\partial t}; R\right]G(x, t) = \delta(x)\delta(t), \qquad [5]$$

where δ denotes the Dirac delta function.

The basic flow is then said to be linearly *stable* if

$$\lim_{t\to\infty} G(x, t) = 0 \quad \text{along all rays} \quad x/t = \text{const};$$

and it is linearly *unstable* if

$$\lim_{t\to\infty} G(x, t) = \infty \quad \text{along some rays } x/t = \text{const}.$$

Among linearly unstable flows, one must further distinguish between two types of impulse response, which are *convectively unstable*, i.e.

$$\lim_{t\to\infty} G(x, t) = 0 \quad \text{along the ray } \frac{x}{t} = 0;$$

and *absolutely unstable*, i.e.

$$\lim_{t\to\infty} G(x, t) = \infty \quad \text{along the ray } \frac{x}{t} = 0.$$

The above can best be realized by examining the x–t diagram given in figure 1(a,b). Disturbances are introduced into the x–t diagram at x_0, all disturbances contained within the wedge are amplified

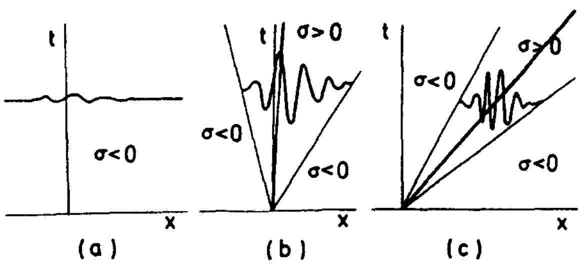

Figure 1. Sketch of impulse responses: (a) stable flow; (b) absolutely unstable flow; (c) convectively unstable flow.

and those outside damped. For the convectively unstable case, as shown in figure 1(a), the amplified disturbances are convected downstream leaving the basic flow undisturbed for a large time. On the other hand, as shown in figure 1(b), for the absolutely unstable case, the amplified disturbances travel both downstream and upstream, and finally contaminate the entire flow field.

In real flows, the non-linearities would prevent the amplitude of the disturbances from becoming unbounded and a saturated amplitude is reached after a certain time. It can be seen that absolutely unstable flows do not require a constant disturbance source, they have been called self-exciting for this reason (Koch 1985).

As one can see, the form of the streamwise perturbation is

$$u = \phi'(y)\exp[i(\alpha_r x - \omega_r t)]\exp(\omega_i t - \alpha_i x). \qquad [6]$$

In the above, $\phi'(y) = d\phi/dy$, $\alpha = \alpha_r + i\alpha_i$ and $\omega = \omega_r + i\omega_i$. The wave growing in the space and time depends on the nature of $\exp(\sigma)$, where

$$\sigma = \omega_i t - \alpha_i x \quad \text{or} \quad \sigma = \left(\omega_i - \alpha_i \frac{x}{t}\right).$$

Each ray in the x–t diagram of figure 1 corresponds to a unique complex wavenumber α^*, where $d\omega(\alpha^*)/d\alpha = x/t$ (Huerre 1987). From figure 1, it is evident that the instability characteristics of the flow can be determined through the ray $x/t = 0$. The wavenumber associated with the ray $x = 0$, called α_0, will satisfy the equation $d\omega(\alpha_0)/d\alpha = 0$, and hence the amplification rate, α, is determined by the relationship, $\omega_i(\alpha_0)$.

According to Bers (1975) and Huerre & Monkewitz (1985), a flow is absolutely unstable if the branch singular point lies in the upper half frequency plane; i.e. $\omega_i(\alpha_0) > 0$. If $\omega_i(\alpha_0) < 0$, then the flow will be convectively unstable or completely stable.

Absolute and convective instability concepts provide the necessary theoretical framework to classify different types of open shear flows according to the qualitative nature of their dynamical behavior. For instance, shear flows that are locally convectively unstable everywhere [e.g. mixing layers (Huerre & Monkewitz 1985)] essentially display extrinsic dynamics. The spatial evolution of the unsteady flow is, in large part, determined by the character (such as amplitude, frequency content etc.) of the excitation that can be tailored to meet specific control goals. Such flows are *noise amplifiers*. On the other hand, as mentioned before, shear flows with absolute instability of sufficiently large size (such as the wake behind a bluff body), may display intrinsic dynamics. These flows behave like *oscillators*.

Mean Flow Profile

Based on the assumptions made in the previous sections, the mean flow velocity profile for the two-phase wake flow is assumed to be identical to that of the particle-free flow. Hence, the profile given by Mattingly & Criminale (1972) for a two-dimensional wake is adopted in our analysis:

$$U^*(y^*) = U^*_\infty + (U^*_c - U^*_\infty)\operatorname{sech}^2(\sigma y^*), \qquad [7]$$

where the asterisk refers to dimensional quantities and U^*_c is the centerline velocity of the wake flow; y^* is the cross-stream coordinate and U^*_∞ is the free-stream velocity.

With the free-stream velocity U^*_∞ as the velocity scale and the half wake width as the length scale, [7] is non-dimensionalized as follows:

$$U(y) = 1 + (U_c - 1)\,\mathrm{sech}^2(\sigma y), \tag{8}$$

where σ is chosen so that at $y = \pm 1$, $(U - 1)/(U_c - 1) = 0.5$. U_c is the dimensionless centerline velocity, which varies with downstream location. The longitudinal variation of U_c is shown in Mattingly & Criminale (1972).

Governing Equations

Assumptions

In addition to the mean flow profile assumption, the following assumptions are also made for the analysis:

1. The flow is incompressible and inviscid.
2. The particles are spheres with diameters which are small compared to the dimensions of large-scale structures.
3. The particles and the flow are in dynamic equilibrium at the beginning of the transient (Wen *et al.* 1988).
4. The material density of the particle is much larger than that of the carrier fluid. (This assumption is valid for gas–solid particle and gas–liquid droplet flows.)
5. The small perturbations imposed on the flow have no effect on the particles during the initial moment.

The governing equations of the two-phase problem consist of two parts: one is for the continuous phase, i.e. the carrier fluid; the other is for the particles. Combining these two parts, one obtains the equation for the stability analysis of the system.

Based on the assumptions made above, one can write the following dimensionless governing equations for the two-phase system.

Continuous phase

Continuity equation,

$$\nabla \cdot \mathbf{u} = 0 \tag{9}$$

Momentum equation,

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = \mathbf{F}_p - \nabla p, \tag{10}$$

where $\mathbf{F}_p$ is the force exerted on the continuous phase by the particles.

Discrete phase

Momentum equation. The general equation for the motion of the particles is the dimensionless BBO equation, which has the following form:

$$\frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} = -\frac{\rho_f}{\rho_p}\nabla p + \frac{f\tau_f}{\tau_A}(\mathbf{u} - \mathbf{u}_p) + \frac{\rho_f}{2\rho_p}(\dot{\mathbf{u}} - \dot{\mathbf{u}}_p) + \frac{3}{\sqrt{2}}\sqrt{\frac{\rho_f\tau_f}{\rho_p\tau_A}}\int_0^t \frac{\dot{\mathbf{u}} - \dot{\mathbf{u}}_p}{\sqrt{t - t'}}\,\mathrm{d}t'. \tag{11}$$

As mentioned in the assumptions, the ratio ρ_f/ρ_p (fluid density/particle density) is $\ll 1$, we may neglect all the terms that are associated with this ratio. Equation [11] is, therefore, reduced to the following:

$$\frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} = \frac{f\tau_f}{\tau_A}(\mathbf{u} - \mathbf{u}_p). \tag{12}$$

The interaction force per unit volume between the continuous phase and the dispersed phase is given by

$$\mathbf{F}_p = \frac{\rho_p' f\tau_f}{\rho_f}\tau_A(\mathbf{u}_p - \mathbf{u}). \tag{13}$$

In the above equation, the bulk density of the fluid is approximated by its material density. The terms that appear in the above equations are defined as follows:

1. $\mathbf{u}_p$ is the dimensionless velocity of the particle; $\mathbf{u}$ is the dimensionless velocity of the flow.
2. ρ_f is the material density of the fluid.
3. ρ'_p is the bulk density of the particles, which is defined as the mass of particles per unit volume of the two-phase mixture; $\rho'_p = N(\rho_p \pi d_p^3/6)$, where N is the particle number density, ρ_p is the particle material density and d_p is the particle diameter.
4. f is the ratio of the actual drag on the particles to the Stokes drag (Crowe *et al.*, 1985).
5. $\tau_A = \rho_p d_p^2/18\mu$ is the particle aerodynamic response time; μ is the dynamic viscosity of the fluid.
6. $\tau_f = L_c/U_\infty$ is the flow characteristic time, where L_c is the length scale of bluff body. Thus, the ratio, τ_A/τ_f, is the Stokes number (St).
7. p is the dimensionless pressure, non-dimensionalized by $\rho_f U_\infty^2$.
8. t, the dimensionless time, is non-dimensionalized by τ_f.

In summary, the dimensionless momentum equations for the two phases are

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = \frac{\rho'_p}{\rho_f}\frac{f\tau_f}{\tau_A}(\mathbf{u}_p - \mathbf{u}) - \nabla p \tag{14}$$

and

$$\frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} = \frac{f\tau_f}{\tau_A}(\mathbf{u} - \mathbf{u}_p). \tag{15}$$

The flow momentum equation can be rewritten as

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = A(\mathbf{u}_p - \mathbf{u}) - \nabla p, \tag{16}$$

where

$$A = \frac{\rho'_p}{\rho_f}\frac{f\tau_f}{\tau_A}. \tag{17}$$

Linearization

Following the traditional small-value perturbation approach explained in detail in Drazin & Reid (1981), one lets

$$\mathbf{u}_p = U(y)\mathbf{i} + \mathbf{u}'(x, y, t), \tag{18}$$

$$p = p_0 + p' \tag{19}$$

and

$$\mathbf{u}_p = U_p(y)\mathbf{i} \tag{20}$$

where the $\mathbf{u}'$ and p' are small perturbations in velocity and pressure, respectively. It is noted that in [20] the fluctuation portion is neglected for the particle phase, in accordance with assumption 5 mentioned previously. Since the solid particles have non-zero aerodynamic response time, their velocities will only begin to react to the fluid perturbation after the fluid transient is started. This paper is concerned with the linear stability of the two-phase flow to an imposed flow perturbation and, therefore, we are only interested in the instantaneous initial stability of the main flow. Subsequent particle velocity response to the flow fluctuation is therefore neglected.

The zeroth-order equation is given as follows after substituting [18]–[20] into [16]:

$$\frac{\mathrm{D}\mathbf{U}}{\mathrm{D}t} = A(\mathbf{U}_p - \mathbf{U}) - \nabla p_0. \tag{21}$$

Since $\nabla p_0 = 0$ for the wake and with the relations $\mathbf{U} = U(y)$, $\mathbf{U}_p = U_p(y)$ and $U_p(y) = U(y)$, [21] is automatically satisfied.

After dropping the higher orders of the small disturbance quantities, the linearized momentum equations for the perturbed velocities reduce to the following form:

$$\frac{\partial u'}{\partial t} + U\frac{\partial u'}{\partial x} + v'\frac{\partial U}{\partial y} = -Au' - \frac{\partial p'}{\partial x} \qquad [22]$$

and

$$\frac{\partial v'}{\partial t} + U\frac{\partial v'}{\partial x} = -Av' - \frac{\partial p'}{\partial y}. \qquad [23]$$

Using a perturbed stream function of the form

$$\psi'(x, y, t) = \phi(y)\exp[i(\alpha x - \omega t)], \qquad [24]$$

the streamwise and the cross-stream perturbed velocity eigenfunctions are related to $\phi(y)$ by

$$u = \frac{\mathrm{d}\phi(y)}{\mathrm{d}y} \qquad [25]$$

and

$$v = -i\alpha\phi(y). \qquad [26]$$

Combining [22] and [23] by removing the pressure terms and substituting in the expressions for velocities yield the following modified Rayleigh equation:

$$(\alpha U - \omega - Ai)\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} - \alpha^2\right)\phi(y) - \alpha\phi(y)U'' = 0, \qquad [27]$$

where $\phi(y)$ is the eigenfunction of this equation.

The solution of this eigenvalue problem, for each value of A, may be viewed as providing a relation between the complex wavenumber α and the complex angular frequency ω. When ω is complex and α is real, the relation provides the temporal instability results. On the other hand, for a real ω and a complex α, the spatial instability can be investigated. In this paper, both will remain complex for the absolute/convective instability study.

Comparing the modified Rayleigh equation for the two-phase flow with the Rayleigh equation of the particle-free flow, one may notice that, under the assumptions made in this analysis, we are able to study the effects of the particles by making a linear transformation of the angular frequency from ω to $\omega - Ai$. Therefore, the presence of particles in the flow always enhances the stability of the flow, because A is a positive quantity.

3. NUMERICAL PROCEDURE

Boundary conditions require that the solution of the modified Rayleigh be bounded for $y \to \pm\infty$. These conditions are denoted here by

$$\mathbf{v}'(\pm\infty) = \mathbf{v}(\pm\infty) = (0, 0), \qquad [28]$$

where the double-argument notation refers to the real and imaginary parts of the complex amplitude, respectively.

When the mean velocity profile is symmetrical with respect to $y = 0$, the modified Rayleigh equation admits both symmetrical and antisymmetrical disturbances. Symmetrical disturbances, which will be referred to as the sinuous mode, satisfy

$$\mathbf{v}(0) = (1, 0) \quad \text{and} \quad \mathbf{v}'(0) = (0, 0); \qquad [29]$$

while antisymmetric disturbances, referred to as the varicose mode, satisfy

$$\mathbf{v}(0) = (0, 0) \quad \text{and} \quad \mathbf{v}'(0) = (1, 0). \qquad [30]$$

Since the varicose mode shows no sign of absolute instability for the mean flow profile given in [8] (Monkewitz 1988) only the symmetric or *Karman* mode is considered in this work. Consequently, boundary conditions [29] are used.

One of the complexities of the wake flow is that U_c is not a constant along the streamwise direction. It varies from one station to another. As shown by Monkewitz (1988), if one travels downstream in the wake, the characteristics of the instability of the flow will change with the location. Both the absolute and the convective instabilities can occur. Therefore, one cannot simply choose the spatial or temporal theory to describe the stability of the wake flow. Both the wavenumber α and frequency ω should be considered as complex numbers.

The modified Rayleigh equation [27] was solved using the shooting method. The numerical integration starts from the centerline ($y = 0$) and marches towards $y = \infty$ where the solution is supposed to match the boundary condition. For the numerical integration of [27], a standard fourth-order Runge–Kutta scheme with constant step size was used. Very intelligent initial guesses for the eigenvalues were important to guarantee the convergence of the algorithm.

The absolute and convective nature of the instability was determined using the Briggs–Bers criterion, which requires the determination of the temporal growth rate of the dominant discrete mode at the location of an impulsive source. From previous discussion, one recalls that for this mode the quantity $d\omega/d\alpha$, which will be referred to as the group velocity, is equal to zero. Hence it corresponds to a saddle point α_0. The corresponding ω_0 is then used to determine the absolute and convective stability nature by examining the imaginary part $\mathcal{I}m(\omega_0)$.

It is not easy to determine the location of the branch point ω_0 or saddle point α_0 computationally, because the entire complex plane is involved. There are several methods suggested by Monkewitz (1988) and by some other authors (Huerre & Monkewitz 1990). The method used in this paper is based on the principle that

$$\frac{d\omega}{d\alpha} = 0 \quad \text{then} \quad \frac{d\omega_r}{d\alpha} = 0 \quad \text{and} \quad \frac{d\omega_i}{d\alpha} = 0. \qquad [31]$$

Numerical differentiation was used to find ω_0 and α_0.

4. RESULTS AND DISCUSSIONS

Sample calculations were performed for the cases of practical interest. The focus in the current analysis is the particle loading parameter, A, defined in [17] as

$$A = af\frac{\rho_p}{\rho_f}\frac{\tau_f}{\tau_A}. \qquad [32]$$

It is noted that in [32] $a = \rho_p'/\rho_p$, where a is actually the local volume fraction of the particles. Therefore, a could be a function of the spatial coordinates to represent different particle distribution patterns. The uniform particle distribution is the simplest case, i.e. $a(y) = \text{const}$. For this case, A is also a constant. As shown in our previous paper (Yang *et al.* 1990), values of A of 0, 0.01 and 0.1 are of practical interest. These values were used in the sample calculations. The case $A = 0$, which corresponds to the particle-free single-phase flow investigated by Monkewitz (1988), provides a verification for our numerical technique and the computer problem. This base case is also useful as a benchmark for establishing the effects of particles. In addition to the uniform particle distribution, two different particle distribution functions were also investigated to assess the effects of particle distribution on the instability of the flow.

Because of the complex nature of the wake flow, both convective and absolute instabilities can exist. Mathematically the presence of particles in the flow can lower the imaginary part of the angular frequency at the saddle point $\mathcal{I}m(\omega_0)$, i.e. the particles can make the flow more convectively unstable or stable. Typical results for the effects of particles on the two-phase stability are shown in figures 2–4. The figures show the map of the wavenumber, α, on the plane of the imaginary part vs the real part of the frequency, ω, at the downstream location $x/L = 0.03$ and the dimensionless centerline velocity $U_c = 0.0012$. The loading parameter, A, varies from 0 to 0.05, and then to 0.1. As α_r increases along the constant α_i lines (labeled $\alpha_i = 0$ or $\alpha_i = -0.4$), curved

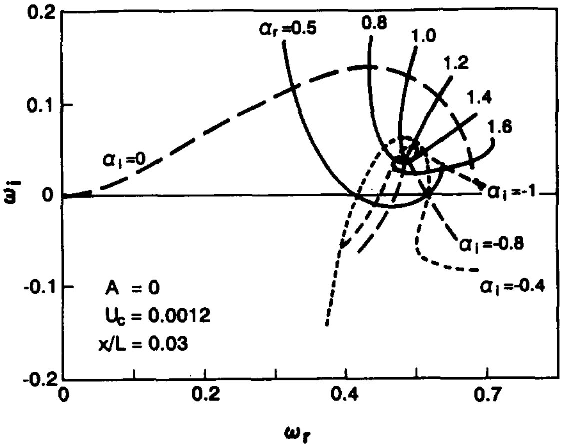

Figure 2. Map of α on the ω plane, for $A = 0$, $U_c = 0.0012$.

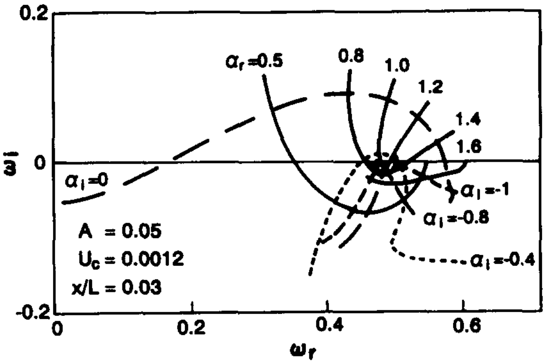

Figure 3. Map of α on the ω plane, for $A = 0.05$, $U_c = 0.0012$ and *uniform particle distributions*.

clockwise contours are traced about the point ω_0, which is the branch point. As A is increased, the imaginary part of the branch point, $\mathscr{I}m(\omega_0)$, decreases from 0.0454786 for $A = 0$ to -0.0045215 for $A = 0.05$ and then to -0.0545217 for $A = 0.1$. This means that the presence of particles in the flow can change the stability characteristics of the flow. The particle-free flow ($A = 0$) is absolutely unstable [$\mathscr{I}m(\omega_0)$ is positive]. The two-phase flow ($A = 0.05$ and 0.1) is convectively unstable [$\mathscr{I}m(\omega_0)$ is negative]. The instability characteristics of the flow switch from the absolute instability to the convective instability due to the presence of particles. It was also found that the variation of $\mathscr{I}m(\omega_0)$ with the particle loading parameter A is linear.

In the neighborhood of the branch point, the relationship, $\alpha = \alpha(\omega)$, is double-valued. With the exception of the neighborhood of this point, lines of constant α_r and α_i are found to intersect orthogonally, indicating the analyticity of the functions $\omega = \omega(\alpha)$ and $\alpha = \alpha(\omega)$. The relationship between ω and α in the neighborhood of the point is clarified by the plot of these same eigenvalue results in the α plane where ω_r and ω_i are constant. This plot, displayed in figure 5, reveals the explicit relationship $\alpha = \alpha(\omega)$. It is clear that α_0 is a saddle point.

The variation of $\mathscr{I}m(\omega_0)$ with the downstream distance x, is shown in figures 6 and 7 for $A = 0$ and 0.05. It is apparent from the figures that the wake flow is absolutely unstable in the upstream near the bluff body, the flow then becomes convectively unstable further downstream. As shown in figure 6, the transition from absolute to convective instability takes place at $x/L = 0.05$, where L is the chord length, which agrees with the data provided by Mattingly & Criminale (1972) for a particle-free wake. This transition point plays a very important role in the stability analysis (Huerre & Monkewitz 1990). Koch (1985) states that the frequency at this point is approximately

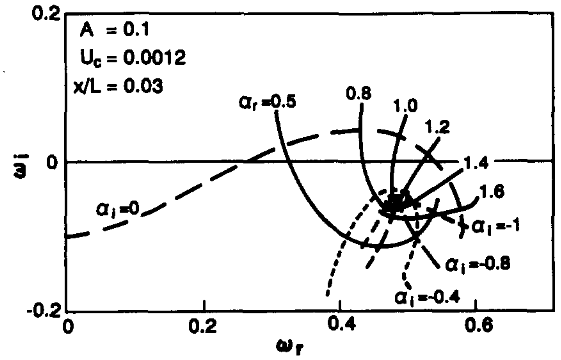

Figure 4. Map of α on the ω plane, for $A = 0.1$, $U_c = 0.0012$ and *uniform particle distributions*.

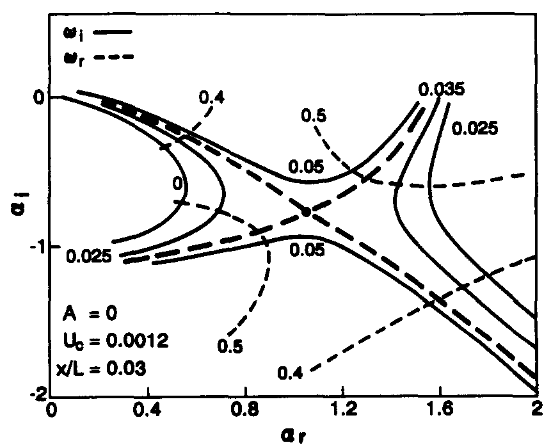

Figure 5. Map of $\alpha(\omega)$ for $A = 0$, $U_c = 0.0012$; the saddle point of this map is located at $\alpha_0 = (1.0886076, -0.7532)$.

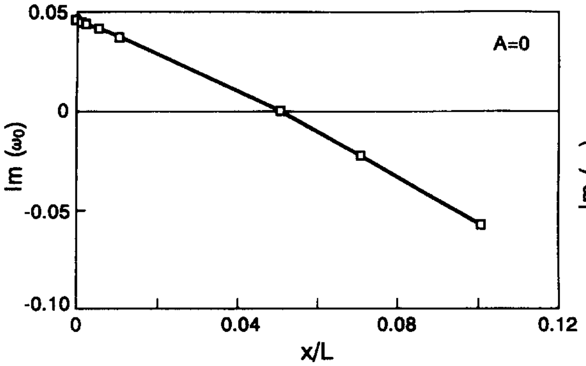

Figure 6. Longitudinal variation of $\mathscr{I}m(\omega_0)$ for $A = 0$.

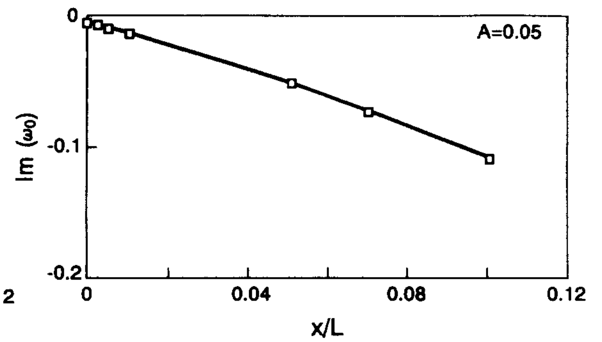

Figure 7. Longitudinal variation of $\mathscr{I}m(\omega_0)$ for $A = 0.05$ and *uniform particle distributions*.

equal to the global-mode frequency of the flow. For the case of a two-phase wake, the result is shown in figure 7. One notes that the location of the transition point is almost at the very beginning of the wake which means that the presence of the particles hastens the transition such that almost the entire wake is convectively unstable. The corresponding frequency variations are shown in figure 8. One finds that the particles lower the frequency in general. This means that the particles lower the global-mode frequency of the wake flow. In other words, the particles slow down the rolling motion of the vortices in the flow, which agrees with the results obtained from the numerical simulation of a two-phase free-shear flow (Tang *et al.* 1989).

Figures 9 and 10 show the variation of $\mathscr{I}m(\omega_0)$ with the parameter A for two different downstream stations. As expected, the larger the A the lower the $\mathscr{I}m(\omega_0)$. The relationship is also linear.

In the above, we presented results for uniform particle distributions. A non-uniform particle distribution is also of practical interest because of its different effects on the wake flow stability. In this paper, we consider a general case, i.e. the particle distribution along the cross-stream direction is represented by a step function, and the loading parameter, A, has the following corresponding form:

$$\left.\begin{array}{lll} A(y) = \mathrm{C} = \text{const} & \text{when} & |y| < D_s \\ A(y) = 0 & \text{when} & |y| > D_s, \end{array}\right\} \qquad [33]$$

where D_s is the half width of the step.

Figure 11 shows the map of α on the ω plane, where the constant C is set at 0.05 and D_s at 1.0. Comparing figure 11 with figure 3, we found that only the $\alpha_i = 0$ line is affected by the step particle distribution. The variations of $\mathscr{I}m(\omega_0)$ with C for $D_s = 1$ are shown together with the uniform

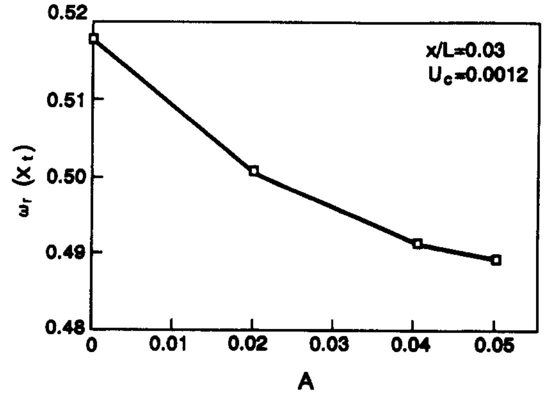

Figure 8. The variation of global mode frequency $\omega_r(x_t)$ with the particle loading parameter A for $U_c = 0.0012$.

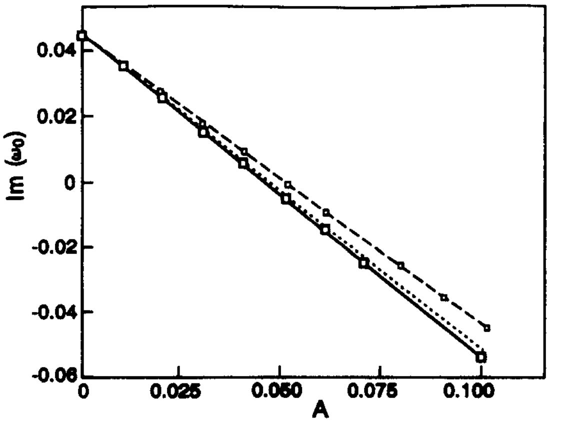

Figure 9. The variation of $\mathscr{I}m(\omega_0)$ with A for $x/L = 0.003$, $D_s = 0.5$ (– – –), 1 (· · ·) and ∞ (——) (*uniform particle distributions*).

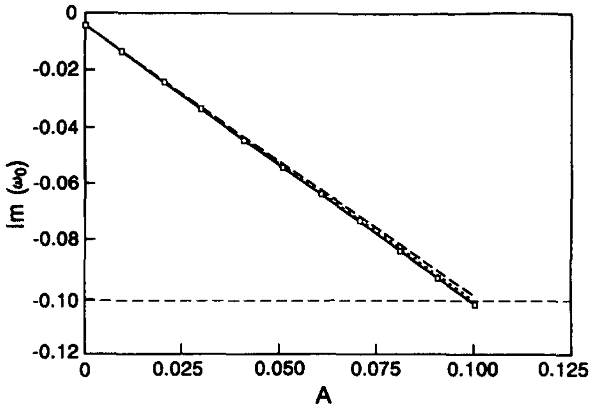

Figure 10. The variation of $\mathscr{I}m(\omega_0)$ with A for $x/L = 0.03$, $D_s = 0.5$ (– – –), 1 (· · ·) and ∞ (——) (*uniform particle distributions*).

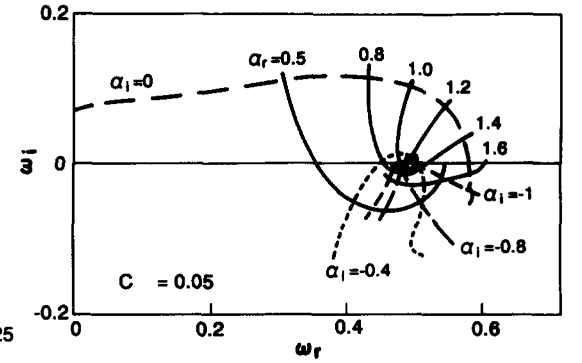

Figure 11. The map of α on the ω plan, for C = 0.05, $D_s = 1$ and $U_c = 0.0012$.

particle distribution case ($D_s = \infty$) in figures 9 and 10 for downstream locations of $x/L = 0.003$ and 0.03, respectively. It is noted that the horizontal coordinate also represents the value of C for the step particle distribution cases. For $D_s = 1$, the results are very close to those of the uniform distribution case. It is important that we learn from this case that those particles located more than the distance L (corresponding to $D_s = 1$) from the centerline have a negligible effect on the wake stability. Also in figures 9 and 10, we plotted the results for $D_s = 0.5$. Comparing these results, one notes that, as expected, the ability of particles to enhance the flow stability is weakened when the particles do not cover the entire flow field. However, the weakening effect due to the narrowing of the wake region occupied by the particles becomes less significant as we move downstream. This is indicated by the fact that the differences are much smaller for $x/L = 0.03$, as shown in figure 10.

To be more specific about the effects of the particle distribution, the slope of the C vs $\mathscr{I}m(\omega_0)$ line is plotted as a function of D_s in figure 12 because of the linear relationship betwen $\mathscr{I}m(\omega_0)$ and C. This figure shows quantitatively that the particle distribution affects the stability characteristics of the wake flow. It is evident that when D_s is small the change is more drastic, which means that the particles are most effective when they are located near the centerline, $y = 0$. This result agrees with the asymptotic nature of [27] which shows that the function $\phi(y)$ behaves like e^{-Dy} with a positive coefficient D for large values of y.

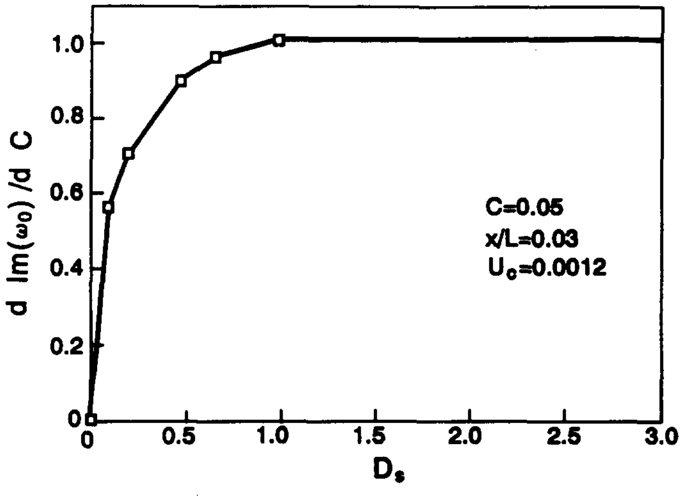

Figure 12. The variation of the slope of $\mathscr{I}m(\omega_0)$ vs C line with D_s for C = 0.05 and $U_c = 0.0012$.

5. CONCLUSIONS

Numerical investigations of the effects of particles on the instability of wake flows are presented. Major simplifying assumptions are that the mean velocity profile of the two-phase flow is identical to that of the single-phase flow and that the particles are initially in dynamic equilibrium with the gas flow. It is also assumed that the continuous-phase flow is incompressible and inviscid. The resulting modified Rayleigh equation was solved for solid particles or droplets in a gas flow.

In general, the presence of particles tends to stabilize the wake flow. An absolutely unstable particle-free wake flow might become convectively unstable when contaminated with particles. The most amplified rates would be lowered due to the particles in a convectively unstable wake flow. It is also found that with the addition of particles into the flow, the global-mode frequency is lower than that of the particle-free case. This means that the particles slow down the rolling motion of large vortices in the flow.

The particle distribution pattern along the cross-stream direction also affects the stability of the wake flow. The most effective particles are those located near the centerline of the wake.

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REFERENCES

BERS, A. 1975 Linear waves and instabilities. In *Physique des Plasmas* (Edited by DEWITT, C. & PEYRAUD, J.), pp. 117–215. Gordon & Breach, New York.

BORHAN, A. & ACRIVOS, A. 1988 The sedimentation of nondilute suspensions in inclined settlers. *Phys. Fluids* **31,** 3488–3501.

BRIGGS, R. J. 1964 Electron stream interaction with plasma. In *MIT Electron Stream Interaction with Plasmas.* MIT Press, Cambridge, MA.

CHIGIER, N. A. 1981 *Energy, Combustion and Environment.* McGraw-Hill, New York.

CHUNG, J. N. & TROUTT, T. R. 1988 Simulation of particle dispersion in an axisymmetric jet. *J. Fluid Mech.* **186,** 199–222.

CROWE, C. T., GORE, R. A. & TROUTT, T. R. 1985 Particle dispersion by coherent structures in free shear flows. *Particle Sci. Technol.* **3,** 149–158.

DRAZIN, P. G. & REID, W. H. 1981 *Hydrodynamic Stability.* Cambridge Univ. Press, London.

HERBOLZHEIMER, E. 1983 Stability of flow during sedimentation in inclined channels. *Phys. Fluids* **26,** 2043–2054.

HO, C. M. & HUERRE, P. 1984 Perturbed free shear layers. *A. Rev. Fluid Mech.* **16,** 365–424.

HUERRE, P. 1987 Spatio-temporal instabilities in closed and open flows. In *Instabilities and Nonequilibrium Structures* (Edited by TIRAPEGUI, E. & VILLARROEL, D.), pp. 141–177. Reidel, Dordrecht.

HUERRE, P. & MONKEWITZ, P. A. 1985 Absolute and convective instabilities in free shear layers. *J. Fluid Mech.* **159,** 151–168.

HUERRE, P. & MONKEWITZ, P. A. 1990 Local and global instabilities in spatially developing flows. *A. Rev. Fluid Mech.* **22,** 473–537.

KAMALU, N., TANG, L., TROUTT, T. R., CHUNG, J. N. & CROWE, C. T. 1989 Particle dispersion in developing shear layers. In *Proc. Int. Conf. on the Mechanics of Two-phase Flows*, Taipei, Taiwan, pp. 199–203.

KOCH, W. J. 1985 Local instability characteristics and frequency determination of self-excited wake flows. *J. Sound Vibr.* **99,** 53–83.

LIFSHITZ, E. M. & PITAEVSKII, L. P. 1981 *Physical Kinetics*, Chap. 6. Pergamon Press, London.

MATTINGLY, G. E. & CRIMINALE, W. O. 1972 The stability of an incompressible two-dimensional wake. *J. Fluid Mech.* **51,** 233–272.

MONKEWITZ, P. A. 1988 The absolute and convective nature of instability in two-dimensional wakes at low Reynolds numbers. *Phys. Fluids* **31,** 999–1006.

SAFFMAN, P. G. 1962 On the stability of laminar flow of a dusty gas. *J. Fluid Mech.* **13,** 120–128.

SHAQFEH, E. S. G. & ACRIVOS, A. 1987 The effects of inertia on the stability of the convective flow in inclined particle settlers. *Phys. Fluids* **30,** 960–973.

TANG, L., CROWE, C. T., CHUNG, J. N. & TROUTT, T.R. 1969 Effect of momentum coupling on the development of shear layers in gas–particle mixtures. In *Proc. Int. Conf. on the Mechanics of Two-phase Flows*, Taipei, Taiwan, pp. 387–391.

WEN, F., KAMALU, N., CROWE, C. Y., TROUTT, T. R. & CHUNG, J. N. 1988 Visualization and measurement of particle dispertion in a turbulent plane mixing layer. In *Proc. 3rd Int. Symp., on Refined Flow Modelling and Turbulence Measurements*, Tokyo, Japan, pp. 765–772.

YANG, Y., CHUNG, J. N., TROUTT, T. R. & CROWE, C. T. 1990 The influence of particles on the spatial stability of two-phase mixing layer. *Phys. Fluids* **A2,** 1839–1845.